SUPERANNUATION AND TIME PAYMENT

KC Notes

Example 1 – without repayments

Mary's grandparents set up a trust fund for her on her first birthday. They contribute $1,000 on each of her birthdays up to and including her 18th birthday. Interest of 8% p.a. is paid annually on the balance of the account one year later. Mary collects the money the day after her 18th birthday. How much does she collect?

Question is asking us to calculate **total money in the bank the day after her 18th birthday**. You're given that **we start with $1,000,** and at the end of every year we **increase 8% of the money** in the bank.

Method 1 – The method that only works with questions without repayments

$A_1 = 1000 \times 1.08^{17}$	First $1000 paid will be affected by 17 increases in interest
$A_2 = 1000 \times 1.08^{16}$	Second $1000 paid affected by 16 increases
…	
$A_{17} = 1000$	Money placed on her 18th birthday (the day before she takes money out)
$Total\ = A_1 + A_2 + A_3 + \cdots + A_{17}$	
$= 1000 \times 1.08^{17} + 1000 \times 1.08^{16} + \cdots + 1000$	
$= 1000\,[1 + 1.08 + 1.08^2 + \cdots + 1.08^{17}]$	Factorising out 1000, note that the [ ] is a geometric series
⇩ $S_n = \frac{a(r^n - 1)}{r - 1} = \frac{1(1.08^{18} - 1)}{1.08 - 1}$	Geometric series, a = 1, r = 1.08, n = 18
$= \$1000 \left[\frac{1.08^{18} - 1}{0.08}\right]$ $= \$37450.24$	

Note: Don't use this method – you'd rather just use one method for all questions. What's that method? Method Two.

Method 2 – The method that works with everything

On her first birthday, $A_1 = 1000$	A_1 here measures the total amount
On her second birthday, $A_2 = 1000 \times 1.08 + 1000$	The \$1000 is paid interest and then parents add more money
On her third birthday, $A_3 = (1000 \times 1.08 + 1000) \times 1.08 + 1000$	The total of last year is paid interest and more money added
$A_3 = 1000 \times 1.08^2 + 1000 \times 1.08 + 1000$ $A_3 = 1000(1 + 1.08 + 1.08^2)$	Simplifying and then factorising the \$1000
$A_n = 1000(1 + 1.08 + 1.08^2 + \cdots + 1.08^n)$ $A_{18} = 1000(1 + 1.08 + 1.08^2 + \cdots + 1.08^{18})$	Note the ( ) is a geometric series
⇩ $S_n = \frac{a(r^n - 1)}{r - 1}$ $= \frac{1(1.08^{18} - 1)}{1.08 - 1}$	Geometric series with a = 1, r = 1.08, n = 18
$A_{18} = \$1000\left[\frac{1.08^{18} - 1}{0.08}\right]$ $= \$37450.24$	

Question 1

a) Jane invests $5,000 at the beginning of each year in a super fund which pays 9% p.a. interest at the end of each year. Jane contributes to the fund for 30 years. How much does she collect at the end of the 30 years?

b) After 10 years, two things happen. The interest rate falls to 6% p.a. and, to compensate, Jane increases her contributions to $7,500 per year. Will she still collect as much as the first scenario? Answer to 2 d.p.

Tip: Find the **balance owing** after 10 years, and then find balance owing after 30 years. Then find the amount she **deposits**, separately. Then **add those two together**.

Example 2 – with repayments

Bill borrows <u>$15,000</u> at 12% p.a. monthly reducible interest, to be repaid over five years with monthly repayments of $M. Find the value of $M.

12% p.a. = 1% per month

Balance owing after 1 month:
$B_1 = 15000 \times 1.01 - M$

The borrowed money is increased in interest, and then Bill repays $M

$B_2 = (15000 \times 1.01 - M) \times 1.01 - M$
$= 15000 \times 1.01^2 - M(1.01 + 1)$

We can simplify it and then factorise out -$M

$B_3 = [(15000 \times 1.01 - M) \times 1.01 - M] \times 1.01 - M$
$= 15000 \times 1.01^3 - M(1.01^2 + 1.01 + 1)$

Do you see a trend?

$B_n = 15000 \times 1.01^n - M(1.01^{n-1} + 1.01^{n-2} + \cdots + 1)$

⇩

$$S_n = \frac{a(r^n - 1)}{r - 1} = \frac{1(1.01^n - 1)}{1.01 - 1}$$

Geometric series with a = 1, r = 1.01

$$B_n = 15000 \times 1.01^n - M\left(\frac{1.01^n - 1}{0.01}\right)$$

When $n = 5 \times 12 = 60$, and $n = 0$:

$$0 = 15000 \times 1.01^{60} - M\left(\frac{1.01^{60} - 1}{0.01}\right)$$

60 months in 5 years, and we want to repay the whole loan by then (n=0)

$$M\left(\frac{1.01^{60} - 1}{0.01}\right) = 15000 \times 1.01^{60}$$

$$M = \frac{15000 \times 1.01^{60} \times 0.01}{1.01^{60} - 1}$$

$M = \$333.67$

Now solve for M (because that's what we're trying to find)

Other variations require you to:

- Find **total time period if repayment amount is given**
- Find total money in **superannuation** account
- Compare time/repayments required when conditions change (i.e. second part of Question 1)

Question 2

Which will give the better financial result after 20 years: a lump sum of $100,000 invested at 6% p.a. compounded annually, or an initial $20,000, followed by monthly payments of $600 with interest at 6% pa. compounded monthly? By how much?

Question 3 (adapted from HSC 2010)

You are Dr Strauss' maths genius. After retirement, Dr Strauss withdraws $2000 at the end of every month from his superannuation account, without making any further deposits. The account continues to earn interest at 0.5% per month.

Let $\$A_n$ be the amount left in the account *n* months after Dr Strauss' retirement, and P the principal amount.

a) Show that $A_n = (P - 400\,000) \times 1.005^n + 400\,000$.

b) If Dr Strauss' account has $200 000, for how many full years after retirement will there be money left in the account?

Question 4 (adapted from HSC 2009)

A year ago, Mr Melser and Dr Strauss borrowed $350,000 to buy an apartment. The interest rate was 9% per annum, compounded monthly. They agree to repay the loan in 25 years with equal monthly repayments of $2,937.

a) Calculate how much money they owe after their first monthly repayment.

b) Strauss and Melser have just made their 12^{th} monthly repayment. They now owe $346,095. The interest rate now decreases to 6% per annum, compounded monthly. The amount, $\$A_n$, owing on the loan after the nth monthly repayment is now calculated using the formula:

$$A_n = 346\,095 \times 1.005^n - 1.005^{n-1}M - \cdots - 1.005M - M$$

where $M is the monthly repayment, and n = 1, 2, …, 288. (Do NOT prove this formula.)

Calculate the monthly repayment if they want the loan to be repaid over the remaining 24 years (288 months).

c) Mr Melser chooses to keep his monthly repayments at $2937. Use the formula in part b) to calculate how long it will take him to repay the $346 095.

d) How much will he save by doing this, rather than reducing his repayments to the amount calculated in part b)?