SUPERANNUATION AND TIME PAYMENT

Quick Answers

1. a) \$742,876 b) \$558,000 2. Second Case is better, by \$22,715.08 3. b) 11
4. a) \$349,688 b) \$2270.31 c) 178.37 months d) \$129,888

Worked Answers

Question 1 a)

$A_1 = 5000 \times 1.09$

$A_2 = (5000 \times 1.09 + 5000) \times 1.09$

$A_3 = [(5000 \times 1.09 + 5000) \times 1.09 + 5000] \times 1.09$

$= 5000 \times 1.09^3 + 5000 \times 1.09^2 + 5000 \times 1.09$

$= 5000(1.09 + 1.09^2 + 1.09^3)$

$A_n = 5000(1.09 + 1.09^2 + 1.09^3 + \cdots + 1.09^n)$

⇩

$$S_n = \frac{1.09(1.09^n - 1)}{1.09 - 1}$$

$$A_n = 5000 \times \frac{1.09(1.09^n - 1)}{0.09}$$

$$A_{30} = 5000\left(\frac{1.09(1.09^{30} - 1)}{0.09}\right)$$

$\boldsymbol{A_{30} = \$742\ 876}$

Question 1 b)

<u>Step 1</u>: Find the balance owing.

$$A_{10} = 5000\left(\frac{1.09(1.09^{10} - 1)}{0.09}\right)$$

$A_{10} = \$82\ 801.50$ – This amount is compounded at 6% p.a. for 20 years.

$\boldsymbol{A_{balance} = 82\ 801.50 \times 1.06^{20} = \$265\ 555.63}$

<u>Step 2</u>: Find the amount of new deposits

$$A_{20} = 7500\left(\frac{1.06(1.06^{20} - 1)}{0.06}\right)$$

$= \$\mathbf{292\ 445}$

<u>Add them together</u> $= \$\mathbf{558\ 000}$

Question 2

<u>First Case</u>:

$A = 100\ 000 \times 1.06^{20} = \$\mathbf{320\ 713.55}$

<u>Second Case</u>:

$Rate\ = 0.06 \div 12 = 0.005$

$A_1 = 20\ 000\ \times 1.005 + 600$

$A_2 = (20\ 000\ \times 1.005 + 600) \times 1.005 + 600$

$A_3 = [(20\ 000\ \times 1.005 + 600) \times 1.005 + 600] \times 1.005 + 600$

$= 20\ 000 \times 1.005^3 + 600(1.005^2 + 1.005^1 + 1)$

$A_n = 20\ 000\ \times 1.005^n + 600(1 + 1.005 + 1.005^2 + \cdots + 1.005^{n-1})$

⇩

$$S_n = \frac{1(1.005^n - 1)}{1.005 - 1}$$

$A_{240} = 20\ 000\ \times 1.005^{240} + 600\left(\frac{(1.005^{240} - 1)}{0.005}\right)$

$A_{240} = \$\mathbf{343\ 428.63}$

Therefore, Second Case is better, by $343\ 428.63 - 320\ 713.55 = \$\mathbf{22\ 715.08}$

Question 3 a)

$Rate = 0.005$

$A_1 = P \times 1.005 - 2000$

$A_2 = (P \times 1.005 - 2000) \times 1.005 - 2000$

$A_3 = [(P \times 1.005 - 2000) \times 1.005 - 2000] \times 1.005 - 2000$

$= P \times 1.005^3 - 2000(1 + 1.005 + 1.005^2)$

$A_n = P \times 1.005^n - 2000(1 + 1.005 + \cdots + 1.005^{n-1})$

⇩

$$S_n = \frac{(1.005^n - 1)}{1.005 - 1}$$

$A_n = P \times 1.005^n - 2000\left(\frac{(1.005^n - 1)}{1.005 - 1}\right)$

$A_n = P \times 1.005^n + \frac{(-2000(1.005^n) + 2000)}{0.005}$

$A_n = P \times 1.005^n + \frac{(-2000(1.005^n) + 2000)}{0.005}$

$$A_n = P \times 1.005^n - \frac{2000(1.005^n)}{0.005} + \frac{2000}{0.005}$$

$$A_n = 1.005^n(P - 400\,000) + 400\,000$$
$$\boldsymbol{A_n = (P - 400\,000) \times 1.005^n + 400\,000}$$

Question 3 b)

$P \; = \$200\,000$

$A_n = 0$

$0 = (200\,000 - 400\,000) \times 1.005^n + 400\,000$

$400000 = 200\,000 \times 1.005^n$

$1.005^n = 2$

$n \log 1.005 = \log 2$

$$n = \frac{\log 2}{\log 1.005}$$

$n = 138.98\, months = 11\, full\ years$

Question 4 a)

$Rate = 0.09 \div 12 = 0.0075$

$A_1 = 350\,000 \times 1.0075 - 2937 = \$\mathbf{349\,688}$

Question 4 b)

$A_n = 346\,095 \times 1.005^n - 1.005^{n-1}M - \cdots - 1.005M - M = 0$

$A_n = 346\,095 \times 1.005^{288} - M(1 + 1.005 + \cdots + 1.005^{287}) = 0$

$$346\,095 \; \times 1.005^{288} - M\left(\frac{(1.005^{288} - 1)}{0.005}\right) = 0$$

$$M = \frac{346\,095 \times 0.005 \times 1.005^{288}}{1.005^{288} - 1} = \$\mathbf{2270.31}$$

Question 4 c)

$$346\,095 \; \times 1.005^n - 2937\left(\frac{(1.005^n - 1)}{0.005}\right) = 0$$

$346\,095 \; \times 1.005^n - 587400 \times 1.005^n + 587400 = 0$

$$1.005^n = \frac{587400}{241305}$$

$$n \log 1.005 = \log\left(\frac{587400}{241305}\right)$$

$n = 178.37\, months$

Question 4 d)

Only consider the payments

Part b) (new repayment) = 288 × 2270 = \$653 760

Part c) (old repayment) = 178.37 × 2937 = \$523 872

Karen saves \$129 888