

Curve Sketching (Asymptotes)

Graphs © Wolfram Alpha LLC

Quick Intro

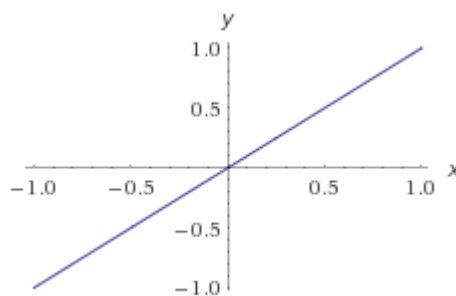
Sketching things by looking at the left and right of asymptotes, and to $\pm\infty$.

Super Quick Easy Example

$$y = x$$

1. There are no asymptotes
2. Test what happens when the graph goes to $\pm\infty$
 - When $x \rightarrow \infty, y \rightarrow \infty$
 - When $x \rightarrow -\infty, y \rightarrow -\infty$

Therefore, you know the graph goes from $-\infty$ to $+\infty$, i.e.



Before We Do A Harder Example

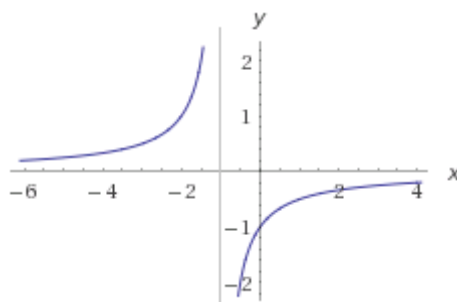
- We (sometimes) use the notation 28^+ and 28^- . It basically means 'a **bit bigger** than 28' and 'a **bit smaller** than 28'.
- So, if there was an asymptote at $x = 0$, we can test $x = 0^+$ and $x = 0^-$ (or something like 0.01 and -0.01) to see where the curve wants to go.

A Harder Example

$$y = -\frac{1}{x+1}$$

1. There are asymptotes:
 - $y(x+1) = -1$, therefore $y \neq 0, x \neq -1$
2. Test what happens when the graph goes to $\pm\infty$
 - When $x \rightarrow \infty, y \rightarrow \frac{-1}{\infty+1} = \frac{-1}{\text{a big number}} = 0^-$ (a bit under the x axis)
 - When $x \rightarrow -\infty, y \rightarrow \frac{-1}{-\infty+1} = \frac{1}{\text{a big number}} = 0^+$ (a bit above the x axis)
3. Test what happens at the asymptote $x \neq -1$
 - When $x \rightarrow -1^+, y \rightarrow \frac{-1}{-1^++1} = \frac{-1}{\text{a -'ve small number}} = \infty$
 - When $x \rightarrow -1^-, y \rightarrow \frac{-1}{-1^-+1} = \frac{-1}{\text{a +'ve small number}} = -\infty$

Therefore, you know that, on the **left**, it goes from 0^+ to $+\infty$... and on the **right**, it goes from $-\infty$ to 0^- , i.e.



Hard Example

Try this: $y = \frac{3x+2}{(2x-1)(x+3)}$

1. Find asymptotes:

When $x \rightarrow -\infty, y \rightarrow$

Test Asymptotes:

When $x \rightarrow -$, $y \rightarrow$

When $x \rightarrow +$, $y \rightarrow$

When $x \rightarrow -$, $y \rightarrow$

When $x \rightarrow +$, $y \rightarrow$

When $x \rightarrow +\infty, y \rightarrow$

Draw Graph.