

3. Risk and Key Cryptography

COMP6441 • KC Notes

3.1 Risk

- Humans struggle to **weigh up risk and put a price on it**
 - Richard could repair his laptop in 2 hours or in 1 hour, but will not think the amount of care and attention behind the curtain – only **cares about the cost**
- We are **not rational** about risk
- You can usually see a bad event occurring, not the chance of a bad event occurring
 - Humans only focus on **the outcome**
 - People are just lucky or unlucky, but in the end **there is the same amount of risk** for each person

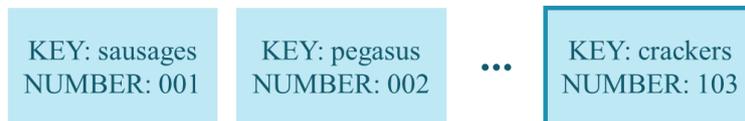
3.2 Low probability, high impact risks

- Humans are particularly bad at measuring risk when **the impact is catastrophic**, but it has a **very low probability** of occurring.
 - We usually shift this risk to another person (e.g. climate change)
- Examples: NASA gathering data on near-Earth objects, volcanoes, mass shootings, the San Andreas fault line, mass shooting
- We are getting good at measuring **risks in finance**, so we should apply ideas from finance
 - Both security and finance **protect something that might happen**
 - Little amount of past data in security, compared to finance
 - Companies struggle to know how much they need to invest in security
- In computing, low probability high impact risks are growing
 - **Companies storing everything in a central location** (single point of failure)
 - AWS, Google Drive, Gmail, Windows
 - This is a trade-off between benefits (speed, costs) and higher impact catastrophes

3.3 Keys: Ralph Merkle

- With a large group of people, each pair of people need a secret key between them
- **Public key cryptography:** a public key for writing, a private key for reading messages
- **Ralph Merkle:**
 - Get a large bag of notes with different keys and a number. Encrypt both message and number, where the owner still knows the original message and number.
 - Share the encrypted messages.
 - Someone can **brute force and solve one note**, gaining one key and one number. They can then also encrypt, but **provide the number (e.g. 103) in plaintext**.
 - The owner can look up the number and decrypt using the key.
 - Third parties will have to **brute force and crack on average half the notes to find note number 103** in order to decrypt.

Bag of notes:



Owner encrypts and shares bag of encrypted notes



Someone chooses one note from the bag **and solves it**.

Then, they can encrypt their own message with this key and send to the owner, keeping the plaintext number.



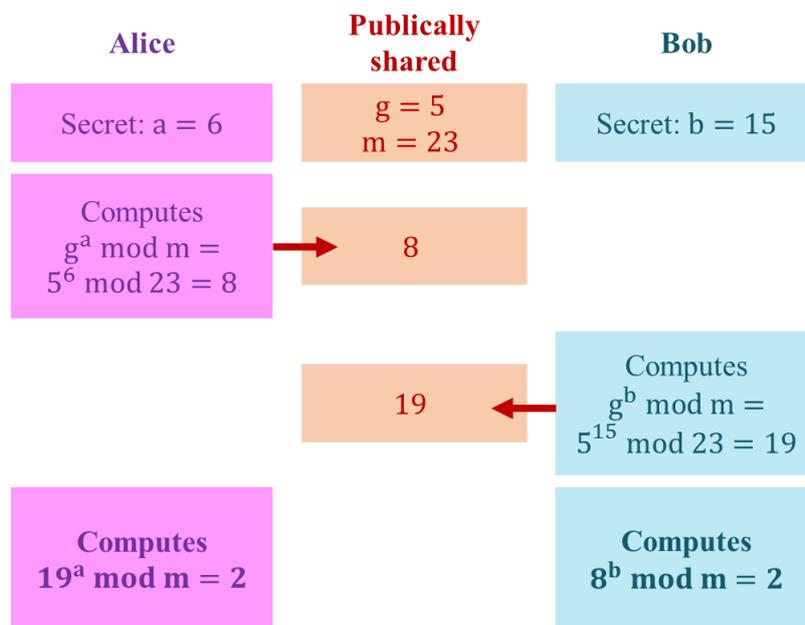
Owner can look up 103, and decrypt since he knows the key is 'crackers'.

Third parties won't know which one is 103, and will have to brute force many of the notes before stumbling on number 103.

- Merkle's system relies on **asymmetry** – it takes much longer to decode every message than to solve one message.

3.4 Keys: Discrete Logarithm Problem and the Diffie-Hellman Key Exchange

- **Discrete Log Problem:** $g^a \bmod m = x$
 - Given g , m and x , **finding a is very difficult.**
 - We can utilise this difficulty to generate a private key
- **Diffie-Hellman Key Exchange:** how can you create a secret key between two people, without it being shared in public?
 1. Alice chooses a secret value **a**.
 2. Bob chooses a secret value **b**.
 3. Alice and Bob decide on public values **g and m**.
 4. Alice sends $g^a \bmod m$ to Bob publicly.
 5. Bob sends $g^b \bmod m$ to Alice publicly.
 6. Alice computes $(g^b \bmod m)^a \bmod m = g^{ab} \bmod m$ using her secret value **a**.
 7. Bob computes $(g^a \bmod m)^b \bmod m = g^{ab} \bmod m$ using his secret value **b**.
 8. Both now have a shared secret key, $g^{ab} \bmod m$!



- It is hard to get a and b , even when given all publically shared numbers.

3.5 Bits of security

- **Bits of security:** a measure of the amount of work needed to decipher a key.
 - Measured in bits (base 2)
- E.g. if we have to do $1000 \approx 2^{10}$ operations (e.g. 1000 'attempts' to decrypt) then it has 10 bits of security
- Exploiting space/time trade-offs (using more than one computer) halves the number of bits
- 128 bits of security is a good ballpark for the universe to end