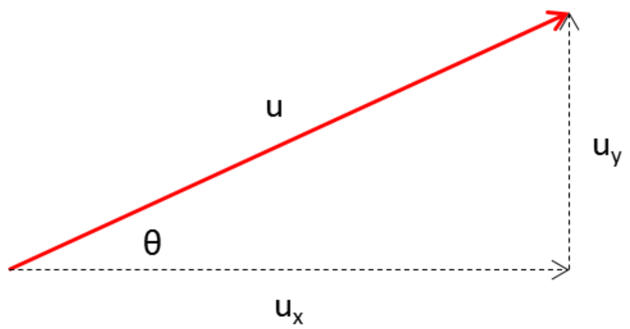
9.2.2 Projectiles, Orbits and Launches

Many factors have to be taken into account to achieve a successful rocket launch, maintain a stable orbit and return to Earth

2.1 Describe the **trajectory of an object** undergoing projectile motion within the Earth's gravitational field in terms of **horizontal and vertical components**

- **Trajectory**: the **path taken** by a projectile
- **Projectile motion**: **motion** under influence of **only weight force** (recall $F = mg$)
  - Composed of **horizontal** and **vertical** motion, both independent of each other
  - Can be expressed as a **vector** – object fired at angle θ with velocity *u*



- Horizontal motion is **constant velocity** (i.e. person throwing sideways)
  - **Velocity u_x = ucosθ**

Equations measuring **horizontal** component

This is because horizontal velocity is **constant**.

$$v_x^2 = u_x^2$$

$$\Delta x = u_x t$$

Where u_x = intial velocity (ms^{-1}), v_x = final velocity (ms^{-1}), Δx = horizontal displacement (m), t = time (s)

- Vertical motion is **constant acceleration** as **gravity** is the only force acting on the object
  - **Velocity u_y = usinθ**

Equations measuring **vertical** component

Acceleration is constant (due to **gravity**).

$$v = u + at$$

$$v_y^2 = u_y^2 + 2a_y\Delta y \ (v^2 = u^2 + 2as)$$

$$y = u_y t + \frac{1}{2}a_y t^2 \ (y = ut + \frac{1}{2}at^2)$$

Where u_y = initial velocity, v_y = final velocity, Δy = vertical displacement, t = time, a_y = acceleration (ms^{-2})

2.2 Describe **Galileo's analysis** of projectile motion

- **Aristotle**'s theory was that **impetus** (air rushing to fill vacuum behind moving object) caused force
- Galileo deduced **parabolic shape** of the trajectory of a projectile
    - Projected a ball from table using an inclined plane
    - Considered having **two independent motions** (horizontal and vertical)

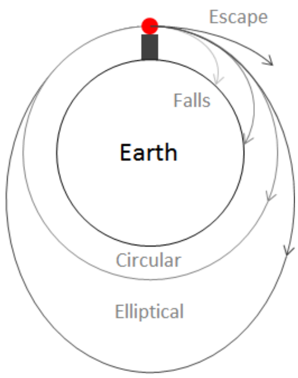
2.3 Explain the concept of **escape velocity** in terms of the **gravitational constant** and **mass and radius** of the planet

- **Escape velocity**: velocity at which an object can rise vertically to **escape from gravitational field** of a planet
    - To escape, **KE must be equal to or higher than E_p (GPE)**
    - Rearranging $KE = \frac{1}{2}mv^2$ being larger or equal to $E_p = G\frac{mM}{r}$ gives:

$$\frac{1}{2}\cancel{m}v^2 \geq G\frac{\cancel{m}M}{r} \quad \rightarrow \quad v \geq \sqrt{\frac{2GM}{r}}$$

Where v = escape velocity (ms^{-1}), G = gravitational constant, M = mass of planet (kg), r = radius (m)

- ***m***, mass of escaping object, **does not affect** escape velocity
    - Therefore, only two influences: **mass and radius** of planet (as mass of object is independent)



2.4 Outline **Newton's** concept of **escape velocity**

- If an object is projected at a height, it will follow a **parabolic path** and land
- Faster: projectile motion will follow a **circular orbit**
- Faster: projectile motion will follow an **elliptical orbit**
- Faster: **parabolic/hyperbolic motion** away from earth and **escape**

2.5 Identify why the term '**g forces**' is used to explain the **forces acting on an astronaut** during launch

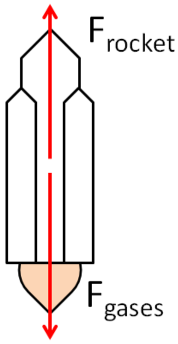
- **g force**: measurement of **acceleration**, measured in **ms^{-2} based on gravity on earth**
    - Normal force is 1g, 9.8ms^{-2} downwards
- Astronaut can feel changes in g-force, and cannot withstand high g forces (4: lose vision/consciousness)
    - Both thrust (upwards) and gravity forces are experienced
    - **Apparent weight**: weight person feels when external force acts on them $= ma + mg$

$$g\ force = \frac{apparent\ weight}{real\ weight} = \frac{mg + ma}{mg}$$

2.6 Discuss the **effect of the Earth's orbital motion** and its **rotational motion** on the launch of a rocket

- Earth spins **west to east**, so launching rocket towards the **east** increases velocity (**465 ms^{-1}**)
    - Launching close to equator increases velocity, e.g. Cape York, Florida
- Earth **orbits the Sun**, so launching **towards Earth's orbit direction** increases velocity (**29.8 kms^{-1}**)

2.7 Analyse the **changing acceleration of a rocket** during launch in terms of the **Law of Conservation of Momentum** and **forces experienced by astronauts**



- **Combustion of hydrogen and oxygen** produces energy, pushes **gases downwards**
- Newton's Third Law: **Law of Conservation of Momentum:** During any **interaction** in a closed system the **total momentum of the system remains unchanged**
- When an object exerts a force on another, it will receive an **equal but opposite force**
  - As gases are pushed out, the rocket is pushed upwards
- **Thrust**: reaction force when mass is ejected in the opposite direction (N)
- Thrust can be measured by **Newton's Second Law, $F = ma$**
  - As force remains constant and mass decreases (gases ejected) **acceleration increases hyperbolically**
- **Multistage rockets**: acceleration becomes close to 0 when section detaches and increases as second stage ignites
- Astronauts similarly experience **changes in acceleration** – force is experienced by astronaut

2.8 Analyse the **forces involved in uniform circular motion** for a range of objects, including **satellites** orbiting the Earth

- **Uniform circular motion**: constant orbital speed but changing orbital velocity
- Change in velocity (due to change in direction) involves acceleration – **centripetal acceleration a_c,** towards centre of circle
  - E.g. a satellite or car

$$a_c = \frac{v^2}{r}$$

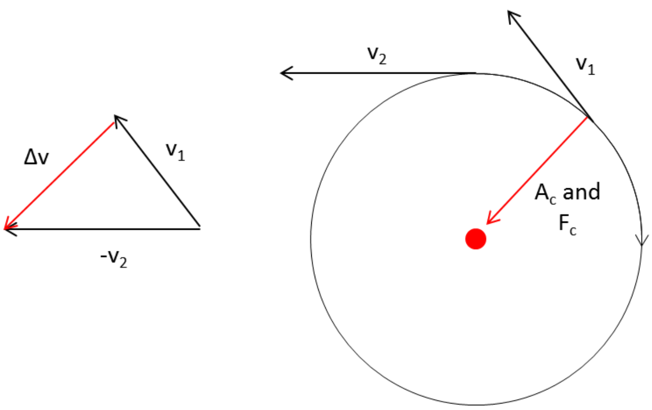
a_c = centripetal acceleration (ms^{-2}), v^2 = linear velocity (ms^{-1}), r = radius (m)

- Acceleration results from a force – **centripetal force F_c,** towards centre of circle
  - E.g. gravitational force between Earth and a satellite, or friction between tyre and road

$$F_c = ma_c = \frac{mv^2}{r}$$

F_c = centripetal force (N), m = mass of object (kg)

- Satellites orbiting Earth has linear orbital velocity through **inertia**, and centripetal force from **gravity**
  - If it **loses inertia** (e.g. hits Earth's atmosphere) it will fall back into Earth



2.9 Compare qualitatively **low Earth** and **geo-stationary orbits**

- **Low Earth orbit**: orbit **radii smaller** and **smaller orbit periods** than geostationary orbits
- **Geo-stationary orbit**: orbit with the same period as Earth – a satellite may look stationary
  - **Must be above** the equator, orbits once every 24 hours

Geo-stationary Orbit	Low Earth Orbit
Altitude: 35,800 km above Earth surface **Period**: 24 hours (equal to Earth periods) Fixed **position** Limited **view** of surface – 1/3	**Altitude**: < 35,800 km above surface **Period**: < 24 hours Variable **position** Able to **view** full surface over several orbits
Does not experience **orbital decay** **Path**: stays the same, easy to track	Atmospheric drag – **orbital decay** occurs **Path**: need to be tracked and controlled to avoid collisions
Uses: communication, weather monitoring, information relay	**Uses**: weather patterns, geo-mapping/scanning

2.10 Define the term **orbital velocity** and the **quantitative** and **qualitative relationship** between orbital velocity, the gravitational constant, mass of the central body, mass of the satellite and the radius of the orbit using **Kepler's Law of Periods**

- **Period (T)**: time taken to complete one orbit
- **Orbital Velocity:** speed to maintain a stable orbit around an object
- **Kepler's Third Law/Law of Periods**: derived in P2.10 and uses $F_c = F_g$ and $v = \frac{circumference}{period}$

$$F_g = F_c \rightarrow = \frac{Gm_1m_2}{r^2} = \frac{m_1v^2}{r}$$

$$v = \sqrt{\frac{GM}{r}} = \frac{2\pi r}{T}$$

Note that m_1 is the mass of the **planet**, so M/m_2 is the mass of the object
(for equating the orbital velocity of a satellite and the velocity based on distance/time). Rearranging will get you:

$$\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$$

Kepler's Third Law

Where r = radius of orbit (m), T = period (s), G = gravitational constant, M = central mass (kg)

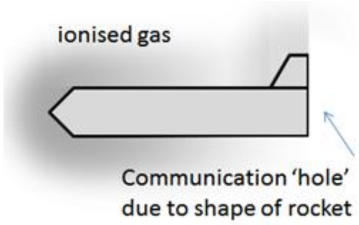
2.11 Account for the **orbital decay of satellites** in **low Earth orbit**

- **Orbital decay**: friction against atmosphere causing atmospheric drag
  - Although density is low, friction will slow orbital velocity and satellite drops to a lower orbit
  - Will either burn in atmosphere and pieces may fall to Earth
- Prevented by attaching **small rocket boosters** to lift satellite

2.12 Discuss **issues associated with safe re-entry** into the Earth's atmosphere and landing on the Earth's surface

SAFE RE-ENTRY

- **Re-entry angle** – if spacecraft enters too steep or shallow it risks damage, see 2.13
- **Heat from friction** – friction with air particles, heat can burn through or melt
  - Heat shields or insulating tiles used to minimise heat
- **G-forces** - astronauts experience high g-forces when the space shuttle decelerates
  - Astronauts my lose sight or consciousness
  - Astronaut should be reclined in a traverse position to keep blood flowing in the brain
- **Ionisation blackout** – particles ionise at the front, blocking radio communication
  - Back of spacecraft has low ionisation due to shape
  - Information can be relayed to satellites and to the spacecraft through the gap

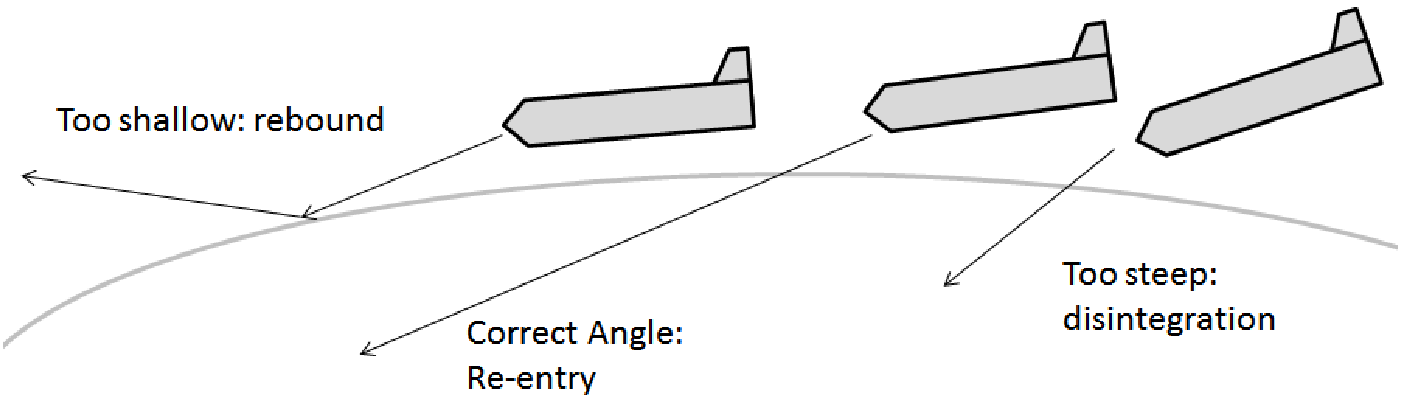


LANDING

- **Weather conditions** - clouds, rain or storms can influence visibility and stability
  - Can also hinder recovery and rescue of spacecraft
  - Prevented by postponing, changing landing sites, decisions made based on weather
- **Landing speed** – spacecraft needs to slow down to prevent breaking parts/safety of astronauts
  - Splashdown through parachuting into water to cushion deceleration
  - Parachute can be used to slow down spacecraft that land on wheels

2.13 Identify that there is an **optimum angle for safe re-entry** or a manned spacecraft into the Earth's atmosphere and the **consequences of failing** to achieve this angle

- Optimum re-entry angle required as **atmosphere causes friction**
  - Lies about 5.2° to 7.2°
- If too **steep**, **friction** between spacecraft and atmosphere is too large and **can cause disintegration**
- If too **shallow**, spacecraft will **bounce off atmosphere** and return to space



2.P1 Solve problems and analyse information to **calculate the actual velocity of a projectile** from its **horizontal and vertical components** using: $v_x^2 = u_x^2$, $v = u + at$, $v_y^2 = u_y^2 + 2a_y\Delta y$, $\Delta x = u_x t$ and $\Delta y = u_y t + \frac{1}{2}a_y t^2$

- **See explanation** of the formulas above.
- In addition, the **maximum height, time of flight (TOF) and range** can be determined
- **Max height** when $v_y = 0$ (i.e. no velocity upwards anymore)
  - Also, for a standard projectile at same height level, occurs **half the range/half the time**
- **Time of Flight** is the time taken until $y = 0$
  - Also, for a standard projectile, is **time taken to cover range/double the horizontal distance covered when reaching max height**
- **Range** until $y = 0$
  - Or, distance covered over TOF/double horizontal distance taken to reach max height
- See Page 23 PiF for examples

2.P2 Perform a first-hand investigation, gather information and analyse data to **calculate initial and final velocity, maximum height reached, range and time of flight of a projectile** for a range of situations by using simulations, data loggers and computer analysis

- **Simulations**: A flash program (PhET, http://phet.colorado.edu/en/simulation/projectile-motion) was used to simulate a cannon firing a cannon ball.
  - **Angle, initial velocity, mass of projectile** could be changed, influencing the **range, height and TOF**
- **First Hand Investigation**: A small ball was rolled from a slanted half-pipe on a table and 'projected' off the table.
  - Initial velocity measured by the distance covered over time on the slanted half-pipe
  - Angle was 90 degrees, as it rolls on the table before projecting off the table into a small pit of sand.
  - Range, time taken to fall and the height of the table could be determined or measured
  - A **data logger** could be used to measure the initial velocity.

2.P3 Identify data sources, gather, analyse and present information **on the contribution** of one of the following to the development of space exploration: Tsiolkovsky, Oberth, **Goddard**, Esnault-Pelterie, O'Neill or von Braun

- **Robert Goddard** (1882 to 1945)
- **Rocket Propulsion in Space**: Proposed that rockets can **fly in vacuums** by applying **Newton's Third Law**
  - Rockets can produce thrust by **pushing gas**
  - Developed a form of **gyroscopic control** for steering a rocket in air
  - Implications: concept of travelling in space, although disputed and contrary to belief
- **Rocket Fuels**: Tested **liquid hydrogen and liquid oxygen** 1909
  - Solid propellants were unsuccessful due to weight, handling and cost
  - Developed **liquid-fuel valving**, and is still used as rocket fuel
- Other Contributions: De Laval's nozzle (heat to power efficiency of 63%), published research, 214 patents

2.P4 Solve problems and analyse information to **calculate the centripetal force acting on a satellite** undergoing uniform circular motion about the Earth using $\boldsymbol{F} = \frac{\boldsymbol{mv^2}}{\boldsymbol{r}}$

2.P5 Solve problems and analyse information using $\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$

- Recall from above. Remember that r is from the **centre of the Earth**